Name: ______________________________________ Date: ___________________

ARITHMETIC AND GEOMETRIC SEQUENCES
COMMON CORE ALGEBRA II

In Common Core Algebra I, you studied two particular sequences known as **arithmetic** (based on constant addition to get the next term) and **geometric** (based on constant multiplying to get the next term). In this lesson, we will review the basics of these two sequences.

ARITHMETIC SEQUENCE RECURSIVE DEFINITION

Given $f(1)$, then $f(n)=f(n-1)+d$ or given a_1 then $a_n=a_{n-1}+d$

where d is called the **common difference** and can be positive or negative.

***Exercise* #1:** Generate the next three terms of the given arithmetic sequences.

(a) $f(n)=f(n-1)+6$ with $f(1)=2$

(b) $a_n=a_{n-1}+\frac{1}{2}$ and $a_1=\frac{3}{2}$

***Exercise* #2:** For some number t, the first three terms of an arithmetic sequence are $2t$, $5t-1$, and $6t+2$. What is the numerical value of the fourth term? Hint: first set up an equation that will solve for t.

It is important to be able to determine a general term of an arithmetic sequence based on the value of the index variable (the subscript). The next exercise walks you through the thinking process involved.

***Exercise* #3:** Consider $a_n=a_{n-1}+3$ with $a_1=5$.

(a) Determine the value of a_2, a_3, and a_4.

(b) How many times was 3 added to 5 in order to produce a_4?

(c) Use your result from part (b) to quickly find the value of a_{50}.

(d) Write a formula for the n^{th} term of an arithmetic sequence, a_n, based on the first term, a_1, d and n.

***Exercise* #4:** Given that $a_1 = 6$ and $a_4 = 18$ are members of an arithmetic sequence, determine the value of a_{20}.

Geometric sequences are defined very similarly to arithmetic, but with a multiplicative constant instead of an additive one.

GEOMETRIC SEQUENCE RECURSIVE DEFINITION

Given $f(1)$ then $f(n) = f(n-1) \cdot r$ or given a_1, then $a_n = a_{n-1} \cdot r$

where r is called the **common ratio** and can be positive or negative and is often fractional.

***Exercise* #5:** Generate the next three terms of the geometric sequences given below.

(a) $a_1 = 4$ and $r = 2$

(b) $f(n) = f(n-1) \cdot \frac{1}{3}$ with $f(1) = 9$

(c) $t_n = t_{n-1} \cdot \sqrt{2}$ with $t_1 = 3\sqrt{2}$

And, like arithmetic, we also need to be able to determine any given term of a geometric sequence based on the first value, the common ratio, and the index.

***Exercise* #6:** Consider $a_1 = 2$ and $a_n = a_{n-1} \cdot 3$.

(a) Generate the value of a_4.

(b) How many times did you need to multiply 2 by 3 in order to find a_4.

(c) Determine the value of a_{10}.

(d) Write a formula for the n^{th} term of a geometric sequence, a_n, based on the first term, a_1, r and n.

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ARITHMETIC AND GEOMETRIC SEQUENCES
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Generate the next **three** terms of each **arithmetic sequence** shown below.

   (a) $a_1 = -2$ and $d = 4$ (b) $f(n) = f(n-1) - 8$ with $f(1) = 10$ (c) $a_1 = 3,\ a_2 = 1$

2. In an arithmetic sequence $t_n = t_{n-1} + 7$. If $t_1 = -5$ determine the values of t_4 and t_{20}. Show the calculations that lead to your answers.

3. If $x+4,\ 2x+5,$ and $4x+3$ represent the first three terms of an arithmetic sequence, then find the value of x. What is the fourth term?

4. If $f(1) = 12$ and $f(n) = f(n-1) - 4$ then which of the following represents the value of $f(40)$?

   (1) -148 (3) -144

   (2) -140 (4) -172 ________

5. In an arithmetic sequence of numbers $a_1 = -4$ and $a_6 = 46$. Which of the following is the value of a_{12}?

   (1) 120 (3) 92

   (2) 146 (4) 106 ________

6. The first term of an arithmetic sequence whose common difference is 7 and whose 22nd term is given by $a_{22} = 143$ is which of the following?

   (1) -25 (3) 7

   (2) -4 (4) 28 ________

7. Generate the next **three** terms of each geometric sequence defined below.

(a) $a_1 = -8$ with $r = -1$ (b) $a_n = a_{n-1} \cdot 3/2$ and $a_1 = 16$ (c) $f(n) = f(n-1) \cdot -2$ and $f(1) = 5$

8. Given that $a_1 = 5$ and $a_2 = 15$ are the first two terms of a geometric sequence, determine the values of a_3 and a_{10}. Show the calculations that lead to your answers.

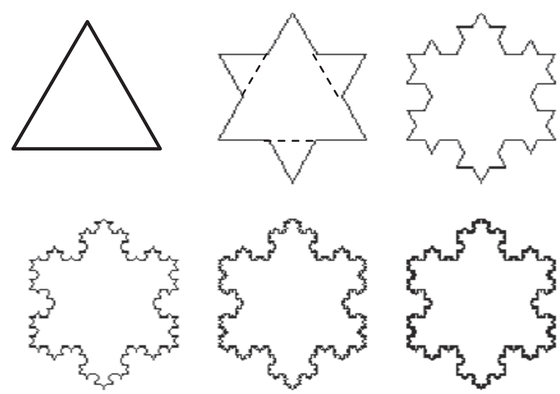
9. In a geometric sequence, it is known that $a_1 = -1$ and $a_4 = 64$. The value of a_{10} is

(1) $-65{,}536$ (3) 512

(2) $262{,}144$ (4) -4096

APPLICATIONS

10. The Koch Snowflake is a mathematical shape known as a **fractal** that has many fascinating properties. It is created by repeatedly forming equilateral triangles off of the sides of other equilateral triangles. Its first six iterations are shown to the right. The perimeters of each of the figures form a geometric sequence.



(a) If the perimeter of the first snowflake (the equilateral triangle) is 3, what is the perimeter of the second snow flake? Note: the dashed lines in the second snowflake are not to be counted towards the perimeter. They are only there to show how the snowflake was constructed.

(b) Given that the perimeters form a geometric sequence, what is the perimeter of the sixth snowflake? Express your answer to the nearest tenth.

(c) If the this process was allowed to continue forever, explain why the perimeter would become infinitely large.